

Noise in Frequency Dividers

Attila Kinali
CISPA Helmholtz Center for Information Security
Saarbrücken, Germany
attila@kinali.ch

Sophie Wenning
Saarland University
Saarland Informatics Campus,
Saarbrücken, Germany
sophie@wenning.org

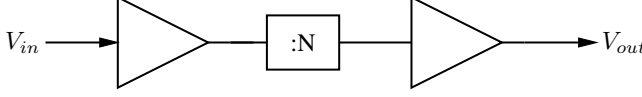


Fig. 1: The circuit model used. The amplifiers are thought to be sine to square-wave converters as in [5, 7].

Abstract—We calculate the noise performance of Π -, Δ -, and Digital Direct Synthesis (DDS) based dividers. Unlike previous calculations that often rely on ad hoc properties of the signal and its transformation, almost always stated without proof and relying on intuition, we start the calculation from the physical behavior of the circuit and derive its performance directly.

Index Terms—frequency division, noise, flicker noise

I. INTRODUCTION

While there is plenty of literature on low noise frequency division implementations, the literature on how to design them, especially how input and circuit noise affects frequency dividers is pretty slim. The most comprehensive work by Egan [1] which is based on his earlier work in [2] gives very good intuition on a lot of the effects seen and his formulas are based on and match experimental results very well. Unfortunately the source of these effects is not rigorously explained. Later works, like those of Calosso and Rubiola [3], who base their work on Egan, run into similar problems.

In this paper we want to give an ab initio explanation of the circuit behavior, especially how noise from various sources gets transformed within the divider. This work can be seen as a continuation of the work presented in [4] and [5] and is thus based as well on the input-sensitivity function formalism of [6]. We will focus on digital frequency dividers, namely simple counters (Π dividers), counters with interpolators (Δ dividers) and DDS based dividers, as they are the most commonly used today.

II. CIRCUIT MODEL

We assume that the input V_{in} is sinusoidal, which we first convert to a square-wave with a (chain of) limiting amplifiers similar to that used in Collins' work [7] and in [5]. The square-wave is then passed through the divider which generates a square-wave output in case of the Π -divider, a triangle-wave in case of the Δ -divider, and a sinusoid in case of the DDS based divider. The output is then passed again through a limiting

amplifier to again generate a square-wave output. This last step is done to ensure that we compare all dividers on a level playing field and do not get artifacts due to variations in the output waveform.

III. CIRCUIT ANALYSIS

A. Previous Results on Amplifier Noise

For completeness, we will quickly repeat the results in [4]. We define phase noise as (c.f. [8, §A.2, pp. 17] and [9, §7.3, pp. 59]):

$$S_{\varphi}(\xi) := \langle \Delta\varphi^2 \rangle_f + (2\pi\nu_0)^2 \langle \Delta t^2 \rangle_f \quad (1)$$

where ν_0 is the signal frequency, $\Delta\varphi$ the variation of the phase of the signal, Δt the variation in delay, and $\langle \Delta\varphi^2 \rangle_f$ and $\langle \Delta t^2 \rangle_f$ are the power-spectral average over unity bandwidth:

$$\langle x \rangle_f := \int_{f-1/2}^{f+1/2} \left| \int_{-\infty}^{\infty} x(t) e^{-2\pi j \zeta t} dt \right| d\zeta \quad (2)$$

Using the impulse sensitivity function (ISF) formalism from Hajimiri and Lee with the resulting phase error due to noise being [6]:

$$\Delta\varphi(t) = \int_{-\infty}^t \Gamma(\tau) n(\tau) d\tau \quad (3)$$

where $\Gamma(t)$ is the ISF and $n(t)$ is the noise that affects the system. In a sine-to-square-wave converter the ISF can be approximated as a comb of alternating positive and negative rectangular pulses, we get the Fourier series representation of (c.f. [4, Formulas 14 and 15])

$$\begin{aligned} \Gamma(t) = & \frac{\tau_w}{T_0} \sum_{k=-\infty}^{\infty} \text{sinc}(\pi k \nu_0 \tau_w) e^{-\pi j k \nu_0 \tau_w} e^{-2\pi j k \nu_0 t} \\ & + \frac{\tau_w}{T_0} \sum_{k=-\infty}^{\infty} \text{sinc}(\pi k \nu_0 \tau_w) e^{-\pi j k \nu_0 \tau_w} e^{-2\pi j k \nu_0 t} e^{-2\pi j k \nu_0 \tau_d} \end{aligned} \quad (4)$$

with ν_0 being the signal frequency, τ_w being the width of the rectangular pulses, τ_d being the phase shift between positive and negative pulses.

Summing over all Fourier components of the ISF leads to a total noise contribution of

$$S_{\varphi, \text{white}}(\xi) \propto \frac{1}{2\pi\nu_0} \langle \Delta\varphi^2 \rangle_f + 2\pi\nu_0 \langle \Delta t^2 \rangle_f \quad (5)$$

for white noise and

$$S_{\varphi, \text{flicker}, \text{single}}(\xi) \propto \langle \Delta\varphi^2 \rangle_f + (2\pi\nu_0)^2 \langle \Delta t^2 \rangle_f \quad (6)$$

$$S_{\varphi, \text{flicker}, \text{multi}}(\xi) \propto 2\pi\nu_0 \langle \Delta\varphi^2 \rangle_f + (2\pi\nu_0)^\alpha \langle \Delta t^2 \rangle_f \quad (7)$$

$$2 \leq \alpha \leq 3 \quad (8)$$

for flicker noise in single-stage limiting amplifiers and multi-stage amplifiers respectively (c.f. [4, Formulas 22, 23 and 26]).

We would like to note, although we give here the scaling of the phase noise proportional to the contributions of phase and delay noise and their scaling with signal frequency, exact values can be computed by plugging in the values of the harmonics of the ISF and summing over them.

B. Noise Scaling in Frequency Dividers

With this preparation we can finally look how noise gets transformed in the divider itself, not accounting for the noise of the output limiting amplifier.

1) *II-Divider*: For the II-divider, we simply drop all but every N -th edge. Which means we transform the ISF such that we now divide ν_0 by N , which we can map to our noise transfer directly, keeping in mind that $\Delta\varphi$ is input related and carries an implicit ν_0 , but we look at the output noise at frequency ν_0/N :

$$S_{\varphi, \text{white}}(\xi) \propto \frac{1}{2\pi\nu_0 N} \langle \Delta\varphi^2 \rangle_f + \frac{2\pi\nu_0}{N} \langle \Delta t^2 \rangle_f \quad (9)$$

$$S_{\varphi, \text{flicker}, \text{single}}(\xi) \propto \frac{1}{N^2} \langle \Delta\varphi^2 \rangle_f + \frac{(2\pi\nu_0)^2}{N^2} \langle \Delta t^2 \rangle_f \quad (10)$$

$$S_{\varphi, \text{flicker}, \text{multi}}(\xi) \propto \frac{2\pi\nu_0}{N^3} \langle \Delta\varphi^2 \rangle_f + \frac{(2\pi\nu_0)^\alpha}{N^\alpha} \langle \Delta t^2 \rangle_f \quad (11)$$

$$2 \leq \alpha \leq 3 \quad (12)$$

We note that we see the $1/N$ or $10\log(N)$ scaling for white noise, as reported in literature (e.g. [2]). The $1/N^2$ or $20\log(N)$ scaling of flicker noise is more interesting. As far as we are aware of, flicker noise scaling in frequency dividers has not been reported. But multiple references, e.g. [2], report that spurs do scale with $20\log(N)$. Flicker noise and spurs do behave similarly, as the are both effects around the carrier and if they are up- and down-converted add up coherently as the harmonics are originating from the same source and thus all harmonics are correlated.

As the output amplifier, in the case of an II-divider, has a gain of 1, as we don't really need an amplifier, at best a buffer for the output.

2) *Λ -Divider and DDS Based Divider*: In the case of the Λ -divider the divider does not drop any of the pulses in the ISF, but scales them. We get first N positive pulses of size $1/N$ and then N negative pulses of size $1/N$. Because we now produce pulses of size $1/N$. This leads to a scaling of the sum of the Fourier components of the ISF by a factor of $1/N$ and thus a scaling of the sampled white noise, additional to the $1/N$ scaling by the reduced frequency. For flicker noise, because of its origin at the baseband and because of its correlated nature, there is no additional gain. The DDS based divider sees the same scaling with ν_0 and N , but due to its output

approximating a sine wave gets its white noise contribution scaled down by $\pi/2$.

$$S_{\varphi, \text{white}}(\xi) \propto \frac{1}{2\pi\nu_0 N^2} \langle \Delta\varphi^2 \rangle_f + \frac{2\pi\nu_0}{N^2} \langle \Delta t^2 \rangle_f \quad (13)$$

$$S_{\varphi, \text{flicker}, \text{single}}(\xi) \propto \frac{1}{N^2} \langle \Delta\varphi^2 \rangle_f + \frac{(2\pi\nu_0)^2}{N^2} \langle \Delta t^2 \rangle_f \quad (14)$$

$$S_{\varphi, \text{flicker}, \text{multi}}(\xi) \propto \frac{2\pi\nu_0}{N^3} \langle \Delta\varphi^2 \rangle_f + \frac{(2\pi\nu_0)^\alpha}{N^\alpha} \langle \Delta t^2 \rangle_f \quad (15)$$

$$2 \leq \alpha \leq 3 \quad (16)$$

IV. CONCLUSION

We calculated the noise performance of three popular types of frequency dividers. We have derived their behavior directly from their physical behavior without resorting to ad hoc properties of the signal's behavior that are stated without proof as was the case, e.g. in [2]. We also added calculations for the behavior of flicker noise, which was ignored in previous publications.

REFERENCES

- [1] W. F. Egan, "Modeling phase noise in frequency dividers," *IEEE Transactions on Ultrasonics, Ferroelectrics, and Frequency Control*, vol. 37, no. 4, pp. 307–315, 7 1990.
- [2] W. Egan, "The effects of small contaminating signals in nonlinear elements used in frequency synthesis and conversion," *Proceedings of the IEEE*, vol. 69, no. 7, pp. 797–811, 1981.
- [3] C. E. Calosso and E. Rubiola, "The sampling theorem in pi and lambda digital frequency dividers," in *2013 Joint European Frequency and Time Forum & International Frequency Control Symposium (EFTF/IFC)*, 2013, pp. 960–962.
- [4] A. Kinali, "A physical sine-to-square converter noise model," in *2018 IEEE International Frequency Control Symposium (IFCS)*, May 2018, pp. 1–6.
- [5] A. Kinali, "A fresh look at the design of low jitter hard limiters," in *2019 Joint Conference of the IEEE International Frequency Control Symposium and European Frequency and Time Forum (EFTF/IFC)*, 2019, pp. 1–4.
- [6] A. Hajimiri and T. H. Lee, "A general theory of phase noise in electrical oscillators," *IEEE Journal of Solid-State Circuits*, vol. 33, no. 2, pp. 179–194, Feb 1998.
- [7] O. Collins, "The design of low jitter hard limiters," *IEEE Transactions on Communications*, vol. 44, no. 5, pp. 601–608, 5 1996.
- [8] E. S. Ferre-Pikal, J. R. Vig, J. C. Camparo, C. R. Ekstrom, C. A. Greenhall, L. Maleki, V. Reinhardt, W. J. Riley, F. L. Walls, and J. D. White, "IEEE standard definitions of physical quantities for fundamental frequency and time metrology—random instabilities," *IEEE Std 1139-2008*, 2 2009.
- [9] A. Kinali, "On time, time synchronization and noise in time measurement systems," Ph.D. dissertation, Max-Planck Institute for Informatics & Saarland University, 2022. [Online]. Available: <http://time.kinali.ch/thesis-final-kinali.pdf>